

Title	数学史は数学教育に役立つか : ICMI Study Meeting, Luminy, France, Apr. 20-25, 1998報告 (数学史の研究)
Author(s)	長岡, 亮介
Citation	数理解析研究所講究録 (1998), 1064: 92-107
Issue Date	1998-10
URL	<a href="http://hdl.handle.net/2433/62445">http://hdl.handle.net/2433/62445</a>
Right	
Type	Departmental Bulletin Paper
Textversion	publisher

## 数学史は数学教育に役立つか —— ICMI Study Meeting, Luminy, France, Apr. 20-25, 1998 報告

長岡 亮介<sup>1</sup>

### 1 ICMI Study

ICMI では、2,000 年に幕張で開かれる ICME-9 に向けて 2 つの Study Book の公刊を計画している。その一つが、

#### On the Role of the History of Mathematics in the Learning and Teaching of Mathematics

であり、これを国際協力で作るための Study Conference が、今回開催されたものである。この集会の IPC(International Programme Committee) の member は

Abraham Arcavi(Israel), Evelyne Barbin (France), Jean-Luc Dorier (France), Florence Fasanelli (USA), John Fauvel (UK, co-chair), Alejandro Garcíadiego (Mexico), Ewa Lakoma (Poland), Jan van Maanen (Netherlands, co-chair), Mogens Niss (Denmark), Man-Keung Siu (Hong Kong)

である。実質 6 日間(月曜の朝から土曜の夜まで!) の conference が、これら IPC member を中心に組織される以下のようなテーマ別の subgroup に分かれて活動することが事前に決まっていた。

本論の最後に参考資料としてつける、筆者の受け取った最初の invitation letter に付属していた研究集会のより詳細な計画にあるように、通常の学会と異なり、IPC で選出した 70 数名に限定した協同作業集会の性格を持った会議である。参加者の選考の詳細に関しては、筆者はまったく知らない<sup>2</sup>。

- **A1** Issues concerning the place of history in educational policy and national curricula
- **A2** Analytical survey of ways of using history in the mathematics classroom
- **A3** Analytical survey of empirical research on classroom delivery & effectiveness of historical dimension: "does using history improve a student's learning and understanding of mathematics, in what respects and how?" kind-of-question

<sup>1</sup>放送大学: 千葉市美浜区若葉 2-11 ryo@u-air.ac.jp

<sup>2</sup>筆者が知る限りでは、Discussion Document に呼応する応募論文などの事前審査による参加者の選考は、必ずしも徹底して行われたわけではなかったようである。筆者は、Luminy meeting の co-chair を勤める John Fauvel 先生(The Open University, UK), Jan van Maanen 先生 (University of Groningen, オランダ) から、ICME-9 の IPC の議長である藤田宏先生への問い合わせにより、藤田先生から推薦され、この meeting に招待を受けたものである。この点は 1998 年 12 月に Singapore で開催される、同じく ICMI Study of *Mathematics Education in the Tertiary level* とは少し様子が違ったようである。

- **A4** Survey of research on necessity/effectiveness of teaching history of mathematics to trainee teachers (initial training and/or in-service training)
- **A5** Analytical survey of research on bearing of history on psychological/conceptual development of students understanding of mathematics
- **A6** Use of history in support of special educational needs
- **B1** The use of original sources in the mathematics classroom
- **B2** Detailed suggestions and examples of use of history in relation to specific subjects (calculus, statistics, &c &c)
- **B3** Philosophical, interdisciplinary and multicultural issues
- **B4** [left unallocated ]
- **B5** History through multimedia, instruments, WWW
- **B6** Bibliographical/resource discussions

Group A は午前、Group B は午後の作業班分類である。

筆者は、Group A1 と Group B5 に所属した。Multimedia の利用をテーマとする Group B5 では convener を勤めた。

実際の会議に先立って、事前の打ち合わせは E-mail を活用して積極的に行われた。その事前準備の周到な成果もあって、会議は、極めて実質的に進行し、最後は友好的な雰囲気の中で成功を祝った。

作業班に別れての discussion のほかに、数回の全体ミーティングと、convener 会議が毎夕開かれた。全体 meeting では、Christian Housel 先生の「地中海数学史構想」を、Siu Man-Keung 先生の「中国 (香港) の数学教育と数学史」、Kahane 先生の「20 世紀数学の総括 — Bourbakism から新しい数学観へ」) などの Plenary Lecture のほかに、group ごとの進行状況の確認が行われた。

## 2 Group A1 の活動

Florence Fasanelli 先生が convener を勤めるこの A1 は当初のやや控え目な “Issues concerning the place of history in educational policy and national curricula” という名称から、“Political framework” というより大きな目標に掲げ替え、数学史を数学教育に活かすためにいかなる戦略をとるべきかという問題提起を視野において、議論が進んだ。主要な参加者は

Rich Millman(Whittier College, California, USA),  
Circe Mary Dynnikov(Brazil)

Gail FitzSimons Gail (Swinburne University of Technology, Australia)  
 Fulvia Furinghetti (Univ di Genova, Italy)  
 Bernard Hodgson (Universite Laval, Canada)  
 Lesley Jones (Goldsmith's University of London, UK)  
 Mogens Niss (Roskilde University, Denmark)  
 Harm Jan Smid (Technical University of Delft, Netherlands)  
 Dianzhou Zhang (East China Normal University, China)  
 Jaime Carvalho e Silva (Univ Coimbra, Portugal)

と筆者で、大きな Working Group であった。

議論の内容は概ね、以下のである。

● 各国の数学史教育の現状報告、相互理解

- 統一カリキュラムのある国 (Spain, Denmark, Italy, Brazil, Litoania, 中国、日本、) と地方自治の国 (Canada, USA-California) の違い
- 教科書の作成方法 (高校現場でつくられる国、一部の著述家の手による規範的な本がつくられる国、膨大な数の教科書会社がありながら大差ない教科書がつくられていく国)
- 数学史の導入状況、とりわけ、教員採用試験の必須科目 (Brazil, 中国) そのもたらず問題点 (教科書不足。教員不足、数学史を講義の中にどのように具体化するかについての具体的な提案の不足)、学生に対する講義に数学史を取り入れている国は依然として少数、一方で教員要請過程での数学史はむしろ常識  $\Rightarrow$  高等師範大学、少なくとも、高校教員の養成課程が、諸外国にはほとんどあるという、考えてみれば当然の事実筆者は驚き、筆者の日本の報告に海外の方たちはびっくり
- 数学教育の必須単位として数学史を入れる根拠
  - \* 数学を教えることが技術的な知識に偏しないように、という National Curriculum において (行政 or それに関わる偉いさんは意外に文化が好き?) 一方に、いわゆる先進国においては進学率の向上による高等教育の危機
  - \* 民族文化主義 ethno-mathematics  $\neq$  国家主義 nationalism
  - \* 愛国主義 patriotism
- 数学史を師範過程に (そして学校教育に) 導入することの困難
  - \* 教材のなさ
  - \* 数学史を学んだことのない教師
  - \* 「普通」との数学との連携
- 数学史を数学教育に取り入れるための政策 (policy)
  - \* 必要性は明らか

- \* されど困難や危険もある
- \* 如何に克服していくか

### 3 Group B5 の活動

ここでは、数学史と数学教育の関係を考える上で、昨今の computer technology に代表される multi-media の有効性、危険性、可能性が論じられるはずであった。しかしながら、multi-media というと computer 上の文字以外の音声・画像情報、とりわけ、近年急速に普及した The Internet 上の WWW などと連携したそれを連想する筆者のような“一般人”に対し、数学教育の世界では、「黒板とチョーク」以外の教育手段を連想する人が少なくなく、discussion ははじめこの問題に集中した。WWW への access 手段さえ、十分でない国々の研究者や、HomePage そのものが新鮮なのか、いろいろな人に熱心に解説している国の人もいて、globalization の時代にあって国際的な地域間格差を目の当たりにする思いをもった。このことがあって、Group B5 の meeting では、WWW に関する議論以上に、

- 学校教育にお行ける数学史から topics をとったドラマ作りの可能性
- 広義の製図道具 (歴史的な装置) を活用した数学教育の可能性
- 数学ソフトウェアを利用した数学史の数学教育への応用可能性

について報告、質疑、そしてこれらと StudyBook の構成への議論がなされた。

いうまでもなく、WWW などの活用は大きなテーマであり、これを group discussion の傘 (umbrella) にたとえる人もいたが、最終的には、技術的な問題を多く孕む topics については Group での discussion を放棄して担当者間で mail でやり取りすることにした。

筆者が、convener を勤めたことも災いしてか、議論が活発であったわりには、後になってみると、鋭い意見対立が噛み合った討論で解決するといった場面はあまりなかったが、member の盛り上がりは十分なものであったようである。

主要な参加者は

Jan van Maanen (University of Groningen, Netherlands)

Karen Dee Michalowicz (USA)

Maria Bussi Bartolini (Università degli Studi di Modena, Italy)

Maria Ponza (Argentina)

磯田 正美 (筑波大学)

Glen Van Brummelen (The King's University College, Canada) (virtual)

そして筆者であった。

**参考資料 1**

## What happens when Mathematical Curricula are composed without any historical Insights?

——— A Consideration about a most significant role of the history of mathematics on teaching of Mathematics

Luminy Meeting of the ICMI Study Group :  
the Role of the History of Mathematics in the teaching and Learning of  
Mathematics

Luminy, Marseilles, France 20-25, April, 1998

**Ryosuke NAGAOKA 長岡 亮介**

Dept. of Mathematics  
The University of the Air

ryo@u-air.ac.jp

## 4 Is the History always a powerful tool in Mathematics?

### 4.1 Mathematicians and Historians

There are a lot of intentions and approaches toward the study of the history of mathematics. Even working mathematicians are sometimes interested in particular topics of history. Intelligence is nothing but an ability to think backward the diverse ground of apparently trivial truths to common sense and thereby to develop a new approach toward unrevealed world. Therefore it is quite natural that intelligent people are interested in the history and intelligent mathematicians are in the history of mathematics whether he is now an active mathematician or a retired person from the front.

But it is important to note that mathematicians' concern for the history is a little bit different from historians. The most serious point is that mathematicians sometimes seek an decisive answer to their historical questions. For example,

- Who is the first discoverer of the complex plane?
- Where can we find the first statement of de Moivre's formula?

- Why did not Newton publish his paper notwithstanding the fact that he had published so many refutative statements against contemporary mathematicians?
- and,so on.

These historical questions are far more difficult to be answered than to be posed. Mathematicians would agree that the more difficult to answer problems are often the more important ones. In the history of mathematics it is just as the case of pure mathematics. And in my view, historical enrichment is nothing but the recognition that even now simplest concepts of mathematics have a vast cultural background and are nothing but a temporarily final form in a long historical developments, which sometimes have proceeded zigzag, not in a straight way.

So, historians like to pose the question in a following way in case of Gaussian plane.

- To which age or to which person's work can we trace an idea to correspond an imaginary number with a geometric figure ,if we at first neglect the theoretical range of the idea?
- On the contrary, in what context did the earlier mathematicians find the theoretical or practical significance of the idea? What problem could they succeed in solving by appealing to it?
- What was their fatal cause which disturbed their further progress of recognition of deeper mathematical significance of it?

These historically interesting problems however seem scarcely attract the concern by most mathematicians.

## 4.2 Teachers and Historians

On another hand, not a few school teachers including university professors are interested in the history with the expectation that historical materials are helpful to encourage students to study mathematics much willingly and eagerly.

It is true that we can find a lot of exciting topics in the history. In fact, diverse kinds of curves can be interestingly lectured if they are exposed in historical perspective, for example in the case of quadratrix.

Even the famous reflective property of a parabola or other quadratic curves might much efficiently be taught in Roberval's style which is not necessarily justified.

Further efforts should be made to "discover" suitable examples that enrich the classroom teaching which is otherwise inclined to be tiresome repletion of exercises or a mere one directional lecture of fundamental knowledge of mathematics.

I do not deny the significant role of historical knowledge in mathematics, but I am of the opinion that the significance is eventual, not to say accidental.

### 4.2.1 Historical Topics is sometimes not a good material.

Let me quote an example, the most famous proof in Euclid's *Elements*.

In a triangle, base angles are equal if sides are equal.<sup>3</sup>

It is a most interesting problem in the history of mathematics why Euclid presented the complex and tedious proof of it. In fact there are several proofs easier than his, although some have a logically fatal defect. A most easiest and clearest way to prove it is to imagine another triangle just up-side-down of the original. As the two sides and an angle between them are equal respectively, the two triangles are congruent. The conclusion is now trivial.

Imagine a teacher deeply indulged in Euclid's *Elements* teaches the theorem to pupils in a classroom in secondary school just in the Euclidian way. I cannot agree with too much optimistic view that this way of teaching makes more and more students like geometry and make them understand the essential spirit of proof in elemental geometry.

I quote another example. One of the most famous story related to the history of mathematics is Pascal's first solution of the problem of the way to divide a betted money between two players when the game had to be paused. I guess that Pascal's solution was really persuasive even to the people in his age and much impressive to us who know elementary probability theory. However it was based only upon an expectation in modern terminology and modern students can solve the problem far easily in a systematic approach by appealing to the formula of Bernoulli's independent trials:

$$P(X = k) = {}_nC_k p^k (1 - p)^{n-k}.$$

### 4.2.2 History are sometimes obliged to be distorted in a classroom.

I fear that there are more serious danger in incorporating history of mathematics in teaching of mathematics. That is the violent simplification/abstraction of historical details, although simplification is a predestinated problem in school teaching.

Let me quote a famous example. Archimedes discovered and rigorously proved that the area of a parabola cut by a line is just  $\frac{4}{3}$  times of the maximal triangle inscribing in it. He exposed his way of discovery in his book *Method*, but it is far from trivial. On the other hand, his published well known proof based upon the summation formula of the geometric progression:

$$1 + \frac{1}{4} + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^3 + \cdots + \left(\frac{1}{4}\right)^{n-1} = \frac{4}{3} \left\{ 1 - \left(\frac{1}{4}\right)^n \right\}$$

is even more difficult. It was a then standard double *reductio ad absurdum* proof, later named as *the method of exhaustion* by Gregoire St. Vincent. Archimedean proof was never based upon a simple limit calculation as

---

<sup>3</sup>English translation may not exact nor correct in a historical sense.



$$\sum_{k=0}^{\infty} \left(\frac{1}{4}\right)^k = \frac{4}{3},$$

which we often find after the 17th Century. I feel St. Vincent's naming itself reflected the view over Greek mathematics at that time.

Now imagine we as a mathematics teacher teach high school students about Archimedean great idea to find the area. If students succeeded in getting the series of areas of inscribed triangles and finding its limit of the sum in the way standard and natural to them, all we can additionally do to guide students is to induce them to make their "proof" more perfect by considering the series of circumscribed triangles and by evaluating the area from both sides. It is never a bad choice for a veteran teacher not to lead high school students to the difficult double *reductio ad absurdum* proof.

If Archimedean way should be taught in High-School classes, one of the essential part of it would be necessarily modified ,extremely simplified or distorted , which is not a history but its disguise and might be worse than a mere fiction.

Of course, my opinion above about introducing Archimedean way does not mean it is always unsuitable in mathematics teaching. In fact, I think the Archimedean method of exhaustion will be extremely efficient when teaching a fundamental concepts and method of higher calculus like  $\epsilon - \delta$  discussion and or the theoretical significance of the so-called Archimedean axiom:

For any positive  $\epsilon$  , and  $L$ , there exists an integer  $N$  such that for any integer  $n > N$ ,  $N\epsilon > L$ .

What I want to make stress on is the difficulties,inefficiency and inadequacy in some cases of incorporating historical raw materials in a standard mathematics courses, NOT the absolute impossibility of it.

## 5 Another look at the possibility of the History of Mathematics

Thus I am not always optimistic to the way history of mathematics is welcomed by mathematicians and pedagogists. But at the same time I want to stress that historical knowledge and sense is very important and essential in Mathematics teaching. I think the important role of history of Mathematics in the teaching of Mathematics can be found in several fields;

- to make clear the extensive meaning or significance of fundamental concepts,like "rational numbers", "axioms", "parabola", "function", "set", etc.

- to present a concrete examples of errors and misunderstanding which former mathematicians really made in developing new methods and concepts.
- to keep in mind the detailed aspect the historical development of mathematical methods and concepts.

I explain each by quoting simple examples.

## 5.1 Historical enrichment in exposure of fundamental concepts

In Japan, irrational number is translated as “無理数” (non-rational or non-reasonable number). It was a mere mistranslation based upon a naive ignorance that European word “rational” has sometimes a meaning “having a ratio” and sometimes “having reason”. But high school students who first come across with  $\sqrt{2}$  are surely embarrassed with the expression “ir-rational” that is “non-reasonable” number. But if teacher is well informed with the Greek discussion on incommensurability, he can lead his students in another way to the irrationality by means of Euclidean algorithm, not in the now familiar way to prove irrationality of  $\sqrt{2}$  by *reductio ad absurdum* proof in the style found in Aristotle’s *Physics*.

I am not arguing that Euclidean Algorithm is better way to introduce irrational numbers. Euclidean method can be less efficient than a standard arithmetic and algebraic proof. But I insist on the view that teachers who are well acquainted with Euclidean Algorithm can give more profound lecture to students. *The important key is not a knowledge of particular historical event or anecdote, but a wide intellectual background of historical development.*

## 5.2 Errors and Misunderstanding by great mathematicians are friends of students who reject to understand Math quickly.

The second point is much related especially to relatively higher mathematics. Ane example to be found in high school mathematics is the concept of imaginary numbers. Historically naive teacher is apt to introduce imaginary numbers by mean of quadratic equation of real coefficients whose discriminant is negative, and he does not feel any uneasiness when his students resist to accept the contradictory notion. All he can do would be to persuade them by appealing to future utility of imaginaries in the field of electro-magnetic science if he is ignorant about the history of imaginaries. But if he is acquainted with the really curious story in which great mathematicians like Charles Babbage and George Peacock were discussing about the reality not only of imaginaries but even also of negatives, he might be able to discover the intellectual possibility or students’ protest. The more he can do with some knowledge about Bombelli’s discussion on Cardano’s formula of a cubic equation.

### 5.3 With historical knowledge we can pause to be a modern being.

In my view the most vital role of the history of mathematics consists in clearly keeping in mind how diverse concepts or methods had been developed. The course of a historical development is one of the most natural way of the development of a human understanding not to say the unique way to be followed at any stage at any occasion. In fact I do not mean that the discussion of conic sections should be taught in the way of Apollonios of Perga before in Cartesian analytic-geometrical way. That idea may be ridiculous in high school teaching. The point that I want to insist is *the vital importance of wide-ranged understanding of the development of human knowledge through historical study*.

In a simplest way I talk here about an historical accident which occurred in the National Curricula of Japanese senior high schools where students from 15 years old to 18 years old are to study for three years.

In Japan, school curricula from elementary schools (primary level) up to senior high schools are strictly controlled by the government, or the Ministry of Education and Culture. And they are to be reformed periodically almost by 10 years. In the latest reformation which was enforced 5 years ago, were carried out a big change of the basic view of the compulsory curriculum of mathematics.

By dividing the subjects of mathematics into two categories; the "cores" (basic standard compulsory to the most) and "the optional" (optional subjects freely chosen). It opened a way of free choice in a national curricula for the first time in Japan although the options were limited to several predecided subjects. But it is therefore at the same time to forbid the traditional way of piling up new mathematical knowledge upon old ones. If some knowledge is indispensable to solve a new problem, students are to get it when they themselves are persuaded with its necessity. For example if they come across with a maximum-minimum problem of a function of degree more than 4, they feel the need to factorize a polynomial of degree more than 3. They will then be well motivated to study the basic factor theorem or remainder theorem of polynomials. For this reason, the basic discussion of formulae including manipulations of complex fractional formulae and the fundamental exercises related to the theory of polynomials are dropped aside from the core curriculum.

Instead, were introduced in core course an elementary probability theory and an elementary discussion of progressions. Someone insisted on the growing importance of finite discrete mathematics in the modern "digital information age" and most mathematicians who are often very naive and never malicious gave an agreement to it.

But all mathematicians and school teachers were really surprised to know that the subject: "numbers and formulae", the most basic part of high school mathematics is put down into one of the optional subjects.

It is true that school teachers are inclined to teach just in the same way as before, and

the traditional systematic way of teaching was not always familiar to learners, although it had been providing logically correct (less erroneous to be exact) and therefore an most efficient way of teaching, but not of learning.

However if the reformers had had a least knowledge of a revolutionary significance of algebraic symbolism discovered and developed in the 17th century and the immense influence range it had on the whole of mathematics, they would have never agreed with the too much daring idea to throw away the fundamental knowledge from the core course.

Not only a method of calculus or a discussion of equation of higher degree, but also the most significant concepts of high school mathematics, for example of a function or a curves can be exposed with their rich contents and in dynamic contexts only in terms of formulae, algebraic symbolism.

An example is enough to prove this proposition. Historians knew very well the first stage of generation of a modern function by Newton, Leibniz, Bernoulli and Euler. The concepts of a function as a pure correspondence once, far afterwards, emerged when Dirichlet challenged to explain the validity of Fourier's series in the case of extremely "discontinuous" function as well.

The rich results of calculus had been developed and accumulated upon the concept of a function as an "*analytical expression*" (*à la Euler*), a formula including variables.

First generation of students who studied under the new curricula have already entered universities. It is a tiresome topics of contemplative communication among professors that even students majoring scientific fields have very little understanding of basic skills and concepts like an equation, and a function, etc, although not a few of them know how to find the derivative and primitive function of a given function. Most teachers suspect the cause of this tendency in the school curricula of mathematics, although this cannot be easily changed without a revolutionary change of the political power. This is a fatal disaster which occurred to recent Japan, in the mathematics education.

Mathematicians and teachers living in today's mathematical paradigm are apt to regard the now standard method (definition and concepts) as the best one. It is very difficult for us, modern being, to dare to abandon the modern standard for the older one. What enables us to pause thinking in modern paradigm is an intellectual modesty back-grounded by profound knowledge of history.

## 参考資料 2

ICMI Study: The role of the history of mathematics in the teaching and learning of mathematics

Introduction to Study Meeting and Study Book

ICMI, the International Commission on Mathematics Instruction, has set up a Study on the role of history of mathematics in the teaching and learning of mathematics. Following the publication of a Discussion Document earlier this year, translated into several languages, there were many expressions of interest from around the world and a large number of contributions to the development of the Study have been submitted. The next stage is a Study Meeting, to be held at Luminy, near Marseilles, France, over the week of April 20-25, 1998. The main purpose of this meeting is to work towards a report, called the Study Book, to be published in time to form part of the agenda at the next International Congress in Mathematics Education (ICME) in Japan in the year 2000.

---

The Study Book

The best way to understand what the Study Meeting is for, how it is to be structured and with what activities, is to consider the eventual product. The ICMI Study Book is to be a major report on the role of history of mathematics in the teaching and learning of mathematics at the end of the twentieth century, to be published in time for presentation in Tokyo in Summer 2000. The functions of the Study Book are to

- % survey and assess the present state of the whole field;
- % provide a resource for teachers and researchers, and for those involved with curriculum development;
- % indicate lines of future research activity;
- % give guidance and information to policy-makers about issues relating to the use of history in pedagogy.

In particular, the Study Book is not a Conference proceedings in any usual sense, and it is not expected that straightforward academic papers or journal-type articles will play a major part of the book. It is not the intention of the co-chairs to simply collect a series of straightforward academic papers or journal-type articles. Discussion and collaboration at the Study Conference will lead to reports which express

common understanding, or commonly felt issues that need further discussion. (The texts submitted so far should be considered as a starting point for this joint work at the Study Conference.)

The following is the draft structure of the Study Book. There will be five major parts to the book, which are given here together with an indication of the principal relevant questions from the Discussion Document (DD), and the relevant working groups (WG), to be explained below.

#### Part 1 Issues of policy/curriculum/social framework

- 1.1 National educational policies (on incorporation of historical dimension in mathematics curriculum) and international comparisons.
- 1.2 Policy issues around the incorporation of history in the training of mathematics teachers.
- 1.3 Role of history in popularizing mathematics in society.
- 1.4 Bearing of recent understandings of the historical development of mathematics (eg in different cultures) on educational & curricular policies.

#### Part 2 Nature of the subject, interdisciplinary and philosophical perspectives [DD #2, #4, #6] [WG A2]

- 2.1 Philosophical issues
- 2.2 Interdisciplinary issues: relations with technics, arts, &c
- 2.3 Social dimensions: survey of recent understandings of historical development of mathematics in cultures (cf 1.4).
- 2.4 Relations between historians of mathematics and mathematics educators

#### Part 3 Educational research and other issues of student framework [DD #1, #10] [WG A3, A4, A5]

- 3.1 Analytical survey of empirical research on classroom delivery & effectiveness of historical perspective/dimension: "does using history improve a student's learning and understanding of mathematics?"  
kind-of-question
- 3.2 Survey of research on necessity/effectiveness of teaching history of mathematics to trainee teachers (initial training and/or in-service training)
- 3.3 Analytical survey of research on bearing of history on psychological/conceptual development of students' understanding of mathematics

#### Part 4 Didactical guidelines and other issues of classroom framework [DD #1, #3, #5, #7, #8, #9] [WG A2, A6, B1, B2]

- 4.1 Analytical survey of ways of using history in the mathematics classroom, and detailed analysis and consideration of some (eg 'original sources')

4.2 Survey of work clarifying the different ages/levels of appropriate historical support

4.3 Survey of work supporting use of history in relation to particular subject areas within the curriculum: 4.3.1 Algebra; 4.3.2 Geometry;

4.3.3 Calculus; 4.3.4 Statistics; &c &c

Part 5 Resources/historiography/bibliography [DD #9, #12] [WG B5, B6]

5.1 Role of history in media support for teaching: computers, CD Roms, films, videos &c

5.2 How to find out more: concrete advice for teachers wanting to become interested in history per se, as well as to find help for historical input/dimension for particular topics (cf 4.3)

5.3 Bibliography: critical/analytical, along the lines described in DD.

[NB These sub-categories are exploratory, and open to fine-tuning in the light of experience and further consideration. The place of something within such a framework says nothing about its significance as an issue --- the classificatory typology carries no evaluative loading: very important issues may be embedded deep within this framework without that carrying any implication for their importance, or the space they need.

There are 17 major sub-parts in the above list at present. Ten or twelve of these will be reflected in the structure of working groups at Luminy (see the following discussion of the Study Meeting). Broadly speaking, each of the five major parts is looking to generate its component of the Book through the activity of one or two working groups.

---

Study Conference April 20-25 1998, CIRM Luminy

The goal of the conference is to create a working forum for the investigation of the theme of the study and to move efficiently towards the production of the Study Book. The following structure has emerged.

- % Each day will contain a balance of plenary and small-group sessions
- % The plenaries will in general be either lectures or panel discussions. Each lecture to be in a one-hour time slot, i.e. 40 minutes plus 15 minutes discussion:
- % The panel discussions will need careful structuring and chairing to ensure they made progress and facilitated proper discussion
- % The small-group sessions would be working groups relating to the structure of the Study Book, as outlined above. These groups

would be responsible for working towards and finding a mechanism for producing the relevant section in the Book.

- % The working groups would themselves be grouped into two classes, A and B: in the morning sessions all the A groups would be working in parallel, in the afternoon sessions all the B groups. Each conference participant will have the opportunity to work in two groups, one from class A and one from class B. These will be decided before the start of the conference.
- % There will be five or six working groups in each of classes A and B, and thus about twelve people in each working group, assuming the conference has around 72 participants.
- % Conference participants are normally invited because they seem likely (from what they submitted in response to the DD, or otherwise) to make a strong contribution to the work of some working group in particular. They are to be asked in the letter of invitation which working group in the other class they would prefer to join. (The organisers will need discretion to ensure roughly equal numbers in each group.)
- % There will be library and resource facilities, to which some thought, care and attention will be paid in advance.
- % The two official conference languages will be French and English, although the eventual Study Book is expected to be in one language.
- % With quite a heavy time-table, there should be a break for relaxation and variety: Thursday April 23rd may be an excursion day, eg to the Chateau D'If.

	Mon April 20	Tue Apr 21	Wed Apr 22	Fri Apr 24	Sat Apr 25
9.00-10.00	Plen. 1	Plen. 3	Plen. 5	Plen. 7	Plen. 9
	(intro)	(lect)	(1/2 way)	(lect)	(panel)
10.30-12.30	WGSA	WGSA	WGSA	WGSA	WGSA
2.30-4.30	WGSB	WGSB	WGSB	WGSB	WGSB
5.00-6.00	Plen. 2	Plen. 4	Plen. 6	Plen. 8	Plen. 10
	(lect)	(panel)	(lect)	(panel)	
	(final)				

- % Three lectures are presently proposed:

Christian Houzel to give a historical survey of mathematics around the Mediterranean (this being the introductory lecture on the Monday).

Karen Parshall to survey recent trends and discoveries in the history of mathematics.



Jean-Pierre Kahane to survey recent developments in mathematics (this being the Wednesday evening lecture).

---

#### Working Groups

Working groups are the main mechanism for producing the report. Each of the following groups will be responsible for the working towards the preparation of one or more sections of the Study Book (some development and refinement of this taxonomy may be expected).

Each participant will be invited to participate in one group (either from the A set or the B set), and is asked to choose one from the other set to work in also. Please let us know your preferences -- noting that the organisers may ask people to participate in another choice if the group sizes look to become unbalanced.

Further guidance on the working of groups will be issued later. For now, please note that the discussion time available for any group is ten hours, a remarkably short time for what is to be achieved! Efficient goal-oriented group activity will be necessary.